

Gravitational radiation from long gamma-ray bursts

Maurice H.P.M. van Putten

Massachusetts Institute of Technology, Cambridge, MA 02139-4307

ABSTRACT

Long gamma-ray bursts (GRBs) are probably powered by high-angular momentum black hole-torus systems in suspended accretion. The torus will radiate gravitational waves as non-axisymmetric instabilities develop. The luminosity in gravitational-wave emissions is expected to compare favorably with the observed isotropic equivalent luminosity in GRB-afterglow emissions. This predicts that long GRBs are potentially the most powerful LIGO/VIRGO burst-sources in the Universe. Their frequency-dynamics is characterized by a horizontal branch in the $\dot{f}(f)$ -diagram.

Subject headings: Gamma-ray bursts: black hole-torus state, gravitational waves

1. Introduction

Cosmological gamma-ray bursts are the most relativistic events in the Universe, at an observed rate of about two per day. The BATSE catalogue shows a bi-modal distribution of GRB durations, with short bursts of about a second and long bursts of about one minute (Kouveliotou et al. 1993; Paciesas et al. 1999). These events are probably associated with the formation of high-angular momentum low-mass black hole-torus systems representing hypernovae (Woosley 1993; Paczyński 1997, 1998; Brown et al. 2000) in star-forming regions (Paczynski 1998; Bloom et al. 2000).

General arguments suggest that the rotational energy in a black hole can be similarly important as accretion, provided there is a channel in place to tap this energy. A maximally rotating black hole of mass $M \sim 7M_\odot$ contains about $2M_\odot$ in rotational energy. Raleigh's criterion (e.g., Kippenhahn & Weigert (1990)) reminds us that rotating systems have a tendency to shed off angular momentum to larger distances, and a rotating black hole is no exception. This may be seen from the first law of black hole thermodynamics. For a black hole of mass M , angular momentum J_H and angular velocity Ω_H we have (Bardeen et al. 1973; Hawking 1975, 1976): $\delta M = \Omega_H \delta J_H + T_H \delta S_H$, where T_H the horizon temperature and S_H ($\delta S_H \geq 0$) the entropy. Thus, the specific angular momentum $a_p = \delta J_H / \delta M$ of a radiated particle exceeds that of the black hole: $a_p \geq 1/\Omega_H \geq 2M > M \geq J_H/M = a$. Angular momentum is stored at a lower cost in radiation than in the black hole, and is emitted with an efficiency of at most 50%. In vacuum, the decay of a Kerr black hole into Schwarzschild black hole is safeguarded by an angular momentum barrier, and spontaneous emission of particles is exponentially small (Teukolsky 1973; Press and Teukolsky 1973; Teukolsky and Press 1974). Yet, black holes formed in hypernovae will be surrounded by magnetized matter,

in the form of an accretion disk or torus. A similar state is expected from tidal break-up of a neutron star by a Kerr black hole (Paczynski 1991; van Putten 1999).

Ultrarelativistic leptonic outflows which form the input to GBRs may be powered by black hole-spin in the presence of magnetic fields (van Putten 2000a,b; Heyl 2000). Long/short GRBs can hereby be associated with magnetic regulated suspended/hyper-accretion onto rapidly/slowly spinning black holes (van Putten & Ostriker 2000). For long bursts, the suspended accretion state lasts for the duration of spin-down of the black hole, whereby the surrounding magnetized matter receives a powerful torque $T = -\dot{J}_H$ from the angular momentum J_H of the black hole (van Putten and Wilson 1999; van Putten 1999; Brown et al. 2001) which arrests the inflow. The black hole performs approximately isotropic work in powering both the outflow and the Maxwell stresses onto the torus. This suggests to consider the possibility that the torus re-radiates this output, in part, in gravitational-wave emissions as it develops non-axisymmetric instabilities.

Here, we show that gravitational wave-emissions are expected in suspended accretion as a collateral feature to long GRB-afterglow emissions. This feature is in many ways similar to new-born pulsars, which are well-known to radiate predominantly in gravitational waves (Shapiro and Teukolsky 1983).

2. Torus in suspended accretion surrounding a rapidly spinning black hole

The approximately isotropic work performed by the black hole takes place over interconnecting magnetic field-lines regulated by the magnetic moment of the black hole in equilibrium with the torus magnetosphere (Wald 1974; Dokuchaev 1987; van Putten 2000b; Lee et al. 2001). These field-lines comprise a torus magnetosphere supported by surrounding baryonic matter and open field-lines to infinity, as schematically indicated in Figure 1. The latter field-lines are endowed with conjugate radiative-radiative boundary conditions, whereas the former have radiative-Dirichlet boundary conditions. These different boundary conditions introduce interactions which will differ in details, e.g.: leptonic outflow to infinity and Maxwell stresses onto the torus, respectively. Nevertheless, the work performed per solid angle into these different field-lines is expected to be rather similar.

The magnetic connection of the black hole to the inner face of the mediates Maxwell stresses by equivalence in poloidal topology to pulsar magnetospheres. An aligned rotator radiates Maxwell stresses to infinity according to $-\dot{J}_p = \Omega_p A_p^2$, where the subscript p refers to the appropriate pulsar values (Goldreich and Julian 1969). Analogously, causal Maxwell stresses are set up between the black hole and the inner face of the torus according to (adapted from Thorne et al. (1986); van Putten & Ostriker (2000))

$$\tau_+ = (\Omega_H - \Omega_T) f_H^2 A^2, \quad (1)$$

where Ω_T denotes the angular velocity of the torus and its similarly shaped force-free magnetosphere, and $2\pi f_H A$ denotes the flux in interconnecting magnetic field-lines - $2\pi A$ representing the

net magnetic flux produced in the torus. This torque received from the black hole compensates for angular momentum losses in magnetic winds and radiation, which arrests the inflow and enables a state of suspended accretion around a rapidly spinning black hole. The suspended accretion state is expected to be stable on average (van Putten & Ostriker 2000).

There powerful shear between the inner and the outer faces of the torus is, to leading order, dictated by Keplerian motion. Some deviation away from Keplerian motion is expected due to the presence of competing torques. They tends to bring the two faces in state of super- and sub-Keplerian motion, with positive radial pressure which promotes a slender shape. The interface separating the two faces is expected to be unstable, which favors turbulent mixing into a state of uniform specific energy across the torus. Mixing enhances differential rotation, as may be illustrated in the Newtonian limit, which gives rise to the angular velocity $\Omega(r) \approx \Omega_K(1 - (r - a)/a)^{1/2}$ as a function of radius r for a torus of major radius a . Compression into a more slender shape tends to reduce differential rotation. The net result should be that the characteristically Keplerian decrease of angular velocity with radius is approximately preserved. The inner and other faces will have, respectively, angular velocities $\Omega_{\pm} \approx \Omega_K(1 \pm 3b/4a)$, where b denotes their radial separation. The same trend should hold in the Kerr metric.

3. Gravitational radiation in suspended accretion

Gravitational radiation from a torus surrounding a black hole tends to dominate radio waves of the same frequency. This is generally due to the compact size in the presence of gravitationally weak magnetic fields. Consider a torus with ellipticity ϵ , a magnetic moment μ_T and mass m in rotation about its center of mass. Its quadrupolar moments in magnetic moment and mass are, respectively, $\epsilon\mu$ and ϵm , which produce luminosities (adapted from Shapiro and Teukolsky (1983)): $\mathcal{L}_{em} \approx \pi^{-1}(\Omega_T M)^4(\mu_T/M^2)^2\epsilon^2$ and $\mathcal{L}_{gw} \approx (32/5)(\Omega_T M)^{10/3}(m/M)^2\epsilon^2$ in geometrical units. These emissions may be compared with, respectively, the luminosity in radio emission $\sim \Omega_p^4 \mu_p^2 / \pi$ from an orthogonal pulsar and in gravitational-wave emissions $\sim (32/5)(\Omega_{orb} \mathcal{M})^{10/3}$ from neutron star-neutron star binaries with angular velocity Ω_{orb} and chirp mass $\mathcal{M} = (M_1 M_2)^{3/5} / (M_1 + M_2)^{1/5}$ (for circular orbits). The ratio of radio-to-gravitational wave emissions can be evaluated as

$$\mathcal{L}_{em} : \mathcal{L}_{gw} \sim (\Omega M)^{2/3} (E_B/M)(M/m)^2 < 1, \quad (2)$$

e.g., when $E_B/M \sim 10^{-6}$ for the relative energy in the magnetic field and $M/m \leq 10^2$.

The fluence F_{GW} in gravitational radiation may be appreciable, provided the torus is long-lived in a state of suspended accretion for the duration of the burst. The suspended accretion state is described by balance of torque and energy:

$$\begin{cases} \tau_+ &= \tau_- + \tau_{rad}, \\ \Omega_+ \tau_+ &= \Omega_- \tau_- + \Omega \tau_{rad} + P_d, \end{cases} \quad (3)$$

where P_d denotes dissipation, $\Omega \approx \Omega_K$ a mean orbital angular frequency and $\tau_- = A^2 f_w^2 \Omega_-$ denotes the torque on the outer face of the torus. The net magnetic flux $2\pi A$ supported by the torus will partially connect to the black hole and to infinity, respectively, with fractions f_H and f_w . Thus, $A \approx ab < B_\theta >$ in terms of the average poloidal component B_θ in the torus. Generally, $f_H + f_w = 1/2 - 1$ with $f_H \propto (M/a)^2$ for a slender torus. A remainder $1 - f_H - f_w$ is inactive in closed field-lines (with Dirichlet-Dirichlet boundary conditions) between the inner light surface and the outer light cylinder, attached to each face as toroidal “bags.” Note that for small differential rotation, we have $(\Omega_K \tau_{rad} + P_d)/\Omega_K \tau_{rad} \approx (\Omega_+ \tau_+ - \Omega_- \tau_-)/\Omega_K (\tau_+ - \tau_-) \approx 2$, in which limit the efficiency of the radiation is 50%.

We shall assume that the coupling between the two faces is dominated by magnetohydrodynamical stresses, due to radial components B_r of the magnetic field. These stresses are dissipative, by Ohmic heating and magnetic reconnection. This will heat the torus, which brings about thermal and, possibly, neutrino emission. By dimensional analysis

$$P_d \approx A_r^2 (\Omega_+ - \Omega_-)^2, \quad A_r = ah < B_r^2 >^{1/2}, \quad (4)$$

where the second equation denotes the root-mean-square of the radial flux averaged over the interface between the two faces with contact area $2\pi ah$. While the angular momentum transport in the shear layer about this interface is mediated by $< B_r^2 >^{1/2}$, the angular momentum transport from the black hole to the torus is mediated by the average $< B_\theta >$. The first comprises the spectral density average over all azimuthal quantum numbers m , whereas the second only involves $m = 0$. Indeed, the net flux through the black hole is generated by the corotating horizon charge $q \approx < B_n > J$ in magnetostatic equilibrium with the mean external poloidal magnetic field (Wald 1974; Dokuchaev 1987; van Putten 2000b; Lee et al. 2001). This averaging process is due to the no-hair theorem. While the exact ratio depends on the details of the magnetohydrodynamic turbulence in the torus, a conservative estimate is that A_r/A is about the square root of the number of azimuthal modes in the approximately uniform infrared spectrum, which should reach up to the first geometrical break at $m = a/b$, i.e.: $A_r/A \approx (a/b)^{1/2}$. Thus, substitution of the first into the second of the stationary conditions (3) gives rise to a positive luminosity

$$\Omega \tau_{rad} \approx \Omega^2 A^2 [3(A_r/A)^2 (b/a) - 2f_w^2] \sim \Omega^2 A^2 \quad (5)$$

in view of $\Omega_- \approx \Omega$, $\Omega_+ - \Omega_- \approx (3/2)(b/a)\Omega$ and $f_w < 1$. The first equation of (3) now reduces to $\Omega/\Omega_H \approx f_H^2/3$. With $\Omega \approx M^{1/2}/R^{3/2}$ and $f_H \propto (M/a)^2$, this shows that $R \propto M^{7/5} \Omega_H^{2/5}$, i.e., the radius of the torus decreases as the black hole spins-down. This defines a horizontal branch of the frequency dynamics in the $\dot{f}(f)$ -diagram (van Putten and Sarkar 2000). At twice the Keplerian frequency of the torus, this produces an observed gravitational wave-frequency of about

$$f_{gw} \sim 1 - 2kH z / (1 + z) \quad (6)$$

for canonical GRB values for a black hole-torus system at redshift z . If the torus is unstable against breaking up in clumps, or if the torus shows violent expansions in its mean radius, the gravitational waves will be episodic, and will correlate with sub-bursts in long GRBs.

The above shows that a stationary state of suspended accretion in the presence of gravitational wave-emissions is facilitated by magnetohydrodynamical viscosity. Note that no specific instability mechanism is identified which is to account for the required non-axisymmetric deformations of the torus. It would be of interest to study this by numerical simulations.

4. Long GRBs as LIGO/VIRGO sources

The GRB-afterglow emissions define the isotropic equivalent luminosity of the black hole in the present black hole-torus model. Given the uniform magnetization of the horizon, the collateral interaction onto the torus is of similar intensity per unit opening angle. It follows that

$$\mathcal{L}_{gw}^{iso} : \mathcal{L}_{grb}^{iso} \sim 1, \quad (7)$$

given that gravitational-wave emission is essentially unbeamed and assuming that the larger fraction of the magnetic field-lines threading the black hole connect to the torus. Long duration continuous emission with the predicted linear chirp is best detected using matched filtering. Taking into account, therefore, the expected gain by a factor \sqrt{n} in sensitivity, where n is the number of cycles in the emission, the effective amplitude of the gravitational radiation at a distance D satisfies

$$h_{eff}^{grb} \sim \left(\frac{M}{D}\right) \left(\frac{F_{GW}}{M}\right)^{1/2} (M\Omega)^{-1/2} (1+z)^{-1} \quad (8)$$

for a net fluence F_{GW} in gravitational waves. With a fraction of order unity of the black hole-luminosity radiated off in gravitational waves, derived from its spin-energy of about one-third its total mass, this points towards GRBs as potentially the most powerful LIGO/VIRGO gravitational-wave burst sources in the Universe. A geometrical beaming factor of 100–200 gives rise to one event per year within a distance $D \sim 100\text{Mpc}$ with $h_{eff} \sim 10^{-20}$. Their approximately monochromatic emissions may have interesting cosmological applications, assuming no cosmological evolution in the GRB parameters.

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Figure captions

Figure 1. Schematic view of the magnetosphere of a black hole-torus system in poloidal cross-section. Shown are the inner and outer torus magnetosphere, attached to the two faces (middle, right) of the torus and the open field-lines to infinity supported by the equilibrium magnetic moment μ_H of the black hole (left). The former have Dirichlet-radiative boundary conditions, the latter have radiative-radiative boundary conditions (D refers to Dirichlet and R/\bar{R} to radiative outgoing/ingoing). The magnetic moment μ_H^e arises in equilibrium with the external magnetic field (van Putten 2000b; Lee et al. 2001) provided by the magnetization μ_T of the torus. It serves to regulate an even horizon flux about its maximum at any rotation rate. A number of inactive field-lines make up inner and outer bags attached to the inner and outer faces of the torus with DD -boundary conditions, which touch an inner and outer light surface (dashed curves). The $D\bar{R}$ - and DR -field lines mediate Maxwell stresses, by equivalence in poloidal topology to pulsar magnetospheres (Goldreich and Julian 1969; van Putten 1999). The strength of the Maxwell stresses on the inner and outer faces corresponds, respectively, to those on a pulsar with angular velocity $-\Omega_{psr} = \Omega_H - \Omega_T$ and $\Omega_{psr} = \Omega_T$, where Ω_H and Ω_T are the angular velocities of the black hole and the torus. This topological equivalence to pulsars also implies causality for the black hole coupling to the inner face. (Reprinted from van Putten (2000b).)

Figure 2. Schematic diagram of the frequency-dynamics trajectory in the $\dot{f}(f)$ -diagram a gravitational wave-signal produced in by a long GRB event from a high-angular momentum black hole-torus system. The gravitational waves are emitted by the torus in suspended accretion. The linear size of the system decreases as the rotational energy of the black hole is radiated away. This produces an approximately horizontal branch on which $\dot{f} \propto f/T$ (right to dashed line) of the twice-Keplerian frequency $1\text{-}2\text{kHz}/(1+z)$ of the torus around a $10M_\odot$ rapidly spinning black hole at redshift z . When the spin of the black hole falls below a critical value, suspended ceases and the torus begins to hyperaccrete onto the black hole. This final phase may be messy, whereby accretion may excite quasi-normal mode ringing (QNR) of the horizon (Papadopoulos & Font 2001). In case of neutron star-black hole coalescence, there exists a well-defined precursor signal, wherein $\dot{f} \propto f^{11/3}$ (left to dashed line). For hypernovae, precursor gravitational wave-emissions are unknown.

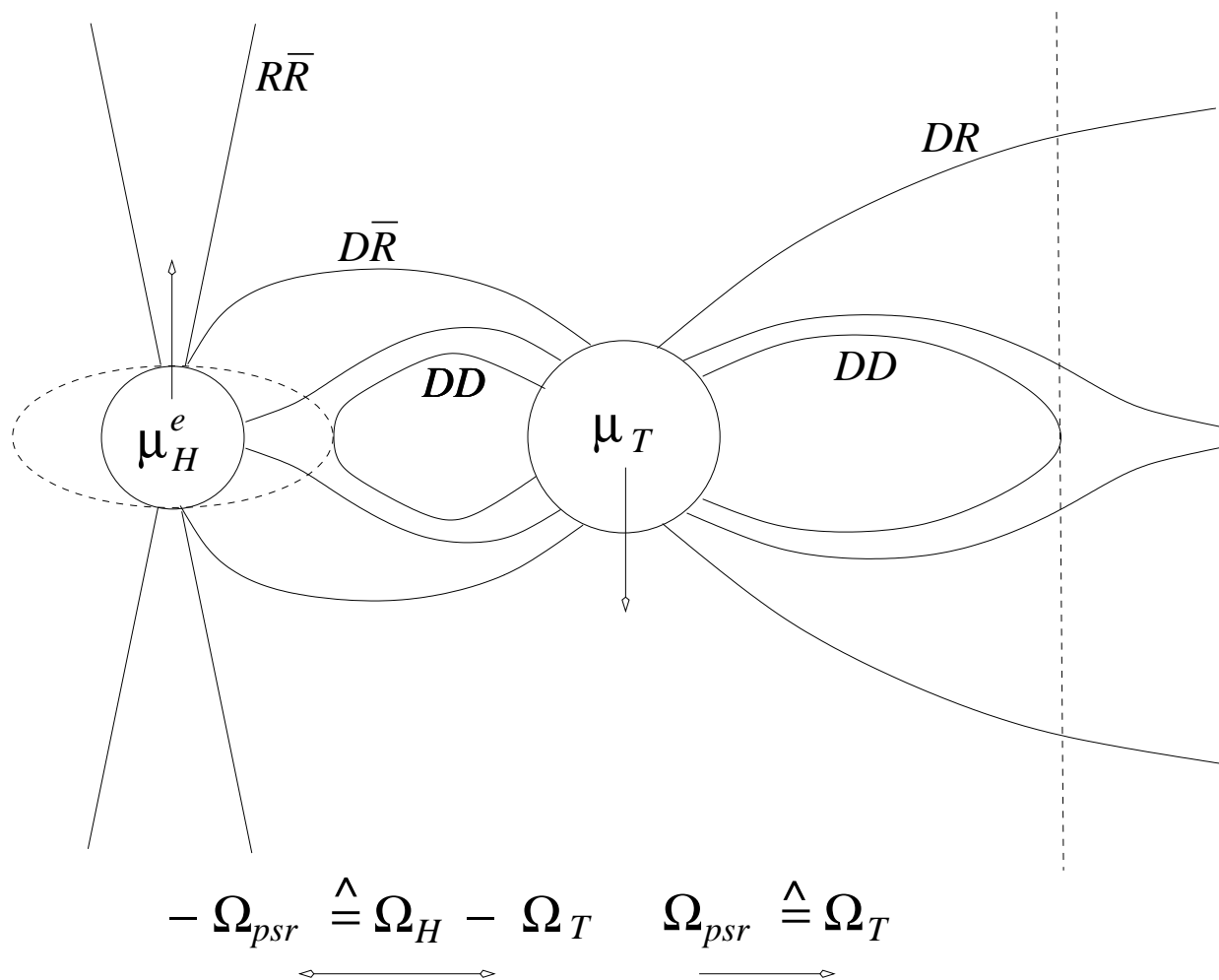


FIGURE 1

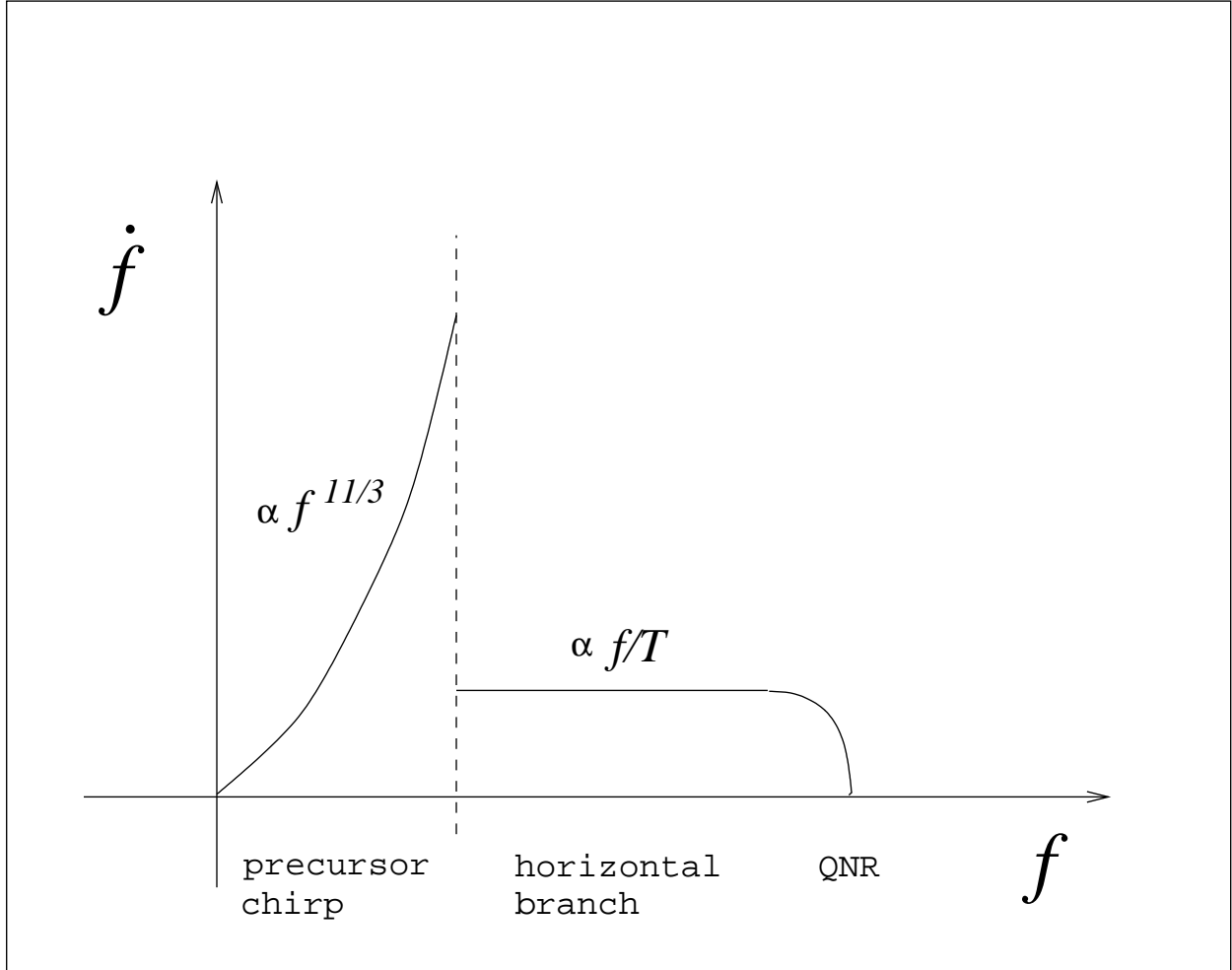


FIGURE 2